Tbilisi Analysis & PDE Seminar



Dear Colleagues!

Institute of Mathematics of the University of Georgia is pleased to invite you to the Online Tbilisi Analysis & PDE Seminar. The seminar is held bi-weekly on Mondays (at 16 : 00 GMT at 17 : 00 CET, at 20 : 00 local time in Tbilisi).

Talk on November 22

Speaker: Vladimir Rabinovich, National Polytechnic Institute of Mexico, ESIME Zacatenco,

https://www.researchgate.net/profile/Vladimir-Rabinovich

The title of the lecture: Interaction problems for the Dirac operators on \mathbb{R}^n

Abstract: We consider the formal Dirac operators on \mathbb{R}^n

$$D_{\boldsymbol{A},\Phi,m,\Gamma\delta_{\Sigma}} = \mathfrak{D}_{\boldsymbol{A},\Phi,m} + \Gamma\delta_{\Sigma} \tag{1}$$

where

$$\mathfrak{D}_{\boldsymbol{A},\Phi,m} = \sum_{j=1}^{n} \alpha_j \left(-i\partial_{x_j} + A_j \right) + m\alpha_{n+1} + \Phi I_N \tag{2}$$

is the Dirac operator on \mathbb{R}^n with a regular magnetic potential $\mathbf{A} = (A_1, ..., A_n)$, electrostatic scalar potentials Φ , and with variable mass m of particles. In formula (2) $\alpha_j, j = 1, ..., n + 1$ are the $N \times N$ Dirac matrices, $N = 2^{\left[\frac{n+1}{2}\right]}, \Gamma \delta_{\Sigma}$ is the singular potential given by $N \times N$ matrix $\Gamma = (\Gamma_{i,j})_{i,j=1}^N$ and δ_{Σ} is the delta-function with the support on a C^2 -hypersurface $\Sigma \subset \mathbb{R}^n$ being the common boundary of open sets $\Omega_{\pm} \subset \mathbb{R}^n$. We associate with the formal Dirac operator (1) the interaction (transmission) problem on \mathbb{R}^n

$$\mathbb{D}_{A,\Phi,m,\mathfrak{B}_{\Sigma}}u = \begin{pmatrix} \mathfrak{D}_{A,\Phi,m}u \text{ on } \mathbb{R}^{n} \setminus \Sigma \\ \mathfrak{B}_{\Sigma}u \text{ on } \Sigma \end{pmatrix}$$
(3)

where $u \to \mathfrak{B}_{\Sigma} u$ is the operator of transmission conditions, generated by the singular potential. We consider the operator $\mathbb{D}_{A,\Phi,m,\mathfrak{B}_{\Sigma}}$ as acting from the Sobolev space $H^1(\Omega_+, \mathbb{C}^N) \oplus H^1(\Omega_-, \mathbb{C}^N)$ into $L^2(\mathbb{R}^n, \mathbb{C}^N) \oplus H^{1/2}(\Sigma, \mathbb{C}^N)$. We introduce the analogues of the Lopatinsky-Shapiro conditions for interaction problems (3) and give the application of these conditions to the self-adjointness of the realization of interaction problem as the unbounded operator $\mathcal{D}_{A,\Phi,m,\mathfrak{B}_{\Sigma}}$ in $L^2(\mathbb{R}^n, \mathbb{C}^N)$. We consider also the Fredholm property of $\mathbb{D}_{A,\Phi,m,\mathfrak{B}_{\Sigma}}$ and the essential spectrum of $\mathcal{D}_{A,\Phi,m,\mathfrak{B}_{\Sigma}}$ for compact and some noncompact C^2 hypersurfaces Σ .

Date: November 22, 2021 **Time:** 16 : 00 GMT (17 : 00 CET and 20 : 00 local time in Tbilisi)

How to join:

The seminar is organized on the **Cisco Webex Meetings**. If you are already registered, you do not need to register again. Otherwise, to join the seminar please send an e-mail to **seminarim@ug.edu.ge** or register here:

https://forms.gle/xfQJ9fg1uqe7CrZw6

You will then receive further information.

WEB of Seminar: https://www.ug.edu.ge/en/tbilisi-analysis-and-pde-seminars

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