

Dear Colleagues!

V. Kupradze Institute of Mathematics of the University of Georgia is pleased to invite you to the Online Tbilisi Analysis & PDE Seminar. The seminar is held bi-weekly on Tuesdays at 20 : 00 local time in Tbilisi.

Talk on February 4, 2025

Speaker: Prof. Gennady Mishuris, Aberystwyth University. https://www.aber.ac.uk/en/maths/staff-profiles/listing/profile/ggm/

"How close are we to solving one of the major problems in matrix factorisation?"

Abstract: The factorisation of arbitrary nonsingular matrix functions on the unit circle remains an open problem in the field of factorisation theory. Current theoretical results are limited to specific classes of matrix functions. Moreover, numerical methods face significant

constraints due to the well-known instability of small perturbations in matrix functions unless the Gohberg-Krein-Bojarski criterion is satisfied. This criterion requires that the difference between the largest and smallest partial indices does not exceed one. Since no general method exists to construct partial indices, this has posed a significant barrier to numerical approaches, severely limiting the practical applications of this otherwise powerful technique.

Recent advancements have addressed these challenges by delivering exact factorisation results for matrix polynomials, effectively overcoming this obstacle [1-2]. Additionally, a robust criterion for identifying stable sets of partial indices has been proposed [3]. In this work, we explore the fundamental ideas underlying this breakthrough and apply them to a class of strictly nonsingular 2×2 matrix functions, demonstrating conditions under which the matrix admits a canonical or stable factorization.

To achieve this, we propose an effective sufficient criterion that ensures that, from a certain point in the approximating series, the given matrix function lies within a small neighbourhood of the stability domain. This theoretical framework relies on an appropriate normalisation of approximate matrix functions. Furthermore, we present numerical results that showcase the effectiveness of the proposed procedure.

Paradoxically, we argue that this numerical approach offers a path to resolving the longstanding factorization problem in its entirety.

L. Ephremidze, I. Spitkovsky. (2020) On explicit Wiener-Hopf factorization of 2×2 matrices in a vicinity of the given matrix. Proc. R. Soc. A 476 (2238): 20200027.

V. M. Adukov, N. V. Adukova, G. Mishuris. (2022) An explicit Wiener-Hopf factorization algorithm for matrix polynomials and its exact realisations within ExactMPF package. Proc. R. Soc. A 478 (2263): 20210941.

N. V. Adukova, V. M. Adukov, G. Mishuris. (2024) An effective criterion for a stable factorisation of strictly nonsin-

gular 2×2 matrix functions. Utilisation of the ExactMPF package. Proc. R. Soc. A 480(2299): https://doi.org/10.1098/rspa.2024.

Date: February 4, 2025

Time: 20 : 00 local time in Tbilisi;

(Compare to your local time: https://www.timeanddate.com/worldclock/georgia/tbilisi);

How to join:

The seminar is organized on the **Zoom Platform**. If you are already registered, you do not need to register again. Otherwise, to join the seminar please send an e-mail to **seminarim@ug.edu.ge** or register here:

https://forms.gle/xfQJ9fg1uqe7CrZw6

You will then receive further information.

WEB of Seminar: https://www.ug.edu.ge/en/tbilisi-analysis-and-pde-seminars

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Organizers:

- 1. R. Duduchava, Institute of Mathematics, University of Georgia, Tbilisi
- 2. E. Shargorodsky, Department of Mathematics, King's College London
- 3. G. Tephnadze, Institute of Mathematics, University of Georgia, Tbilisi

Secretary:

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