



Dear Colleagues!

Institute of Mathematics of the University of Georgia is pleased to invite you to the Online Tbilisi Analysis & PDE Seminar. The seminar is held bi-weekly on Mondays (at 16 : 00 GMT at 17 : 00 CET, at 20 : 00 local time in Tbilisi).

**Talk on January 17, 2022**

**Speaker:** Nikolai L. Vasilevski, Department of Mathematics, CINVESTAV, Mexico City, Mexico  
<https://www.math.cinvestav.mx/~nvasilev/>

**The title of the lecture:** **On analytic type function spaces and direct sum decomposition of  $L_2(D, d\nu)$**

**Abstract:** Let  $D$  be either the unit disk  $\mathbb{D}$  or  $\mathbb{C}$ , and let  $J$  be either  $[0, 1)$  or  $\mathbb{R}_+$ , so that  $D = J \times \mathbb{T}$ , where  $\mathbb{T}$  is the unit circle in  $\mathbb{C}$ . We set  $\mathcal{H}$  for any weighted Hilbert space  $L_2(D, d\nu)$ , with the probability measure  $d\nu(z) = \omega(|z|)dA(z)$ , where  $dA(z) = \frac{1}{\pi}dx dy$ ,  $z = x + iy$ , and whose radial weight function  $\omega : D \rightarrow \mathbb{R}_+$  is such that the linear span of the monomials  $z^p \bar{z}^q$ , for all  $p, q \in \mathbb{Z}_+$ , is dense in  $\mathcal{H}$ .

Given any pair  $(m, n) \in \mathbb{Z}_+ \setminus \{(0, 0)\}$ , we denote by  $\mathcal{A}^{(m, n)}$  the subspace of  $\mathcal{H}$ , which consists of all smooth functions  $f$  satisfying the equation  $\frac{\partial^m}{\partial z^m} \frac{\partial^n}{\partial \bar{z}^n} f = 0$ , and by  $\mathcal{A}_k^{(m, n)}$  the subspace of  $\mathcal{H}$ , which consists of all smooth functions satisfying the equation  $\left(\frac{\partial^m}{\partial z^m} \frac{\partial^n}{\partial \bar{z}^n}\right)^k f = 0$ . We call such functions  $(m, n)$ -analytic, and  $k$ -( $m, n$ )-polyanalytic, respectively.

For particular values of  $(m, n)$ , we have already known spaces of

- analytic functions  $\mathcal{A} = \mathcal{A}^{(0, 1)}$ ,
- $k$ -polyanalytic functions  $\mathcal{A}_k = \mathcal{A}^{(0, k)}$ ,
- anti-polyanalytic functions  $\tilde{\mathcal{A}} = \mathcal{A}^{(1, 0)}$ ,
- $k$ -anti-polyanalytic functions  $\tilde{\mathcal{A}}_k = \mathcal{A}^{(k, 0)}$ ,
- harmonic functions  $\mathcal{H} = \mathcal{A}^{(1, 1)}$ ,
- $k$ -polyharmonic functions  $\mathcal{H}_k = \mathcal{A}^{(k, k)}$ .

We develop a unified approach to the characterization of all these analytic type function spaces and prove, in particular, the following result.

Given any predefined "analytic quality of functions",  $(m, n) \in \mathbb{Z}_+ \setminus \{(0, 0)\}$ , the Hilbert space  $L_2(D, d\nu)$  admits the following direct sum decomposition

$$L_2(D, d\nu) = \bigoplus_{k \in \mathbb{N}} \mathcal{A}_{(k)}^{(m, n)},$$

where  $\mathcal{A}_{(k)}^{(m, n)} = \mathcal{A}_k^{(m, n)} \ominus \mathcal{A}_{k-1}^{(m, n)}$  are the spaces of the so-called true- $k$ -( $m, n$ )-polyanalytic functions.

**Date:** January 17, 2022

**Time:** 16 : 00 GMT (17 : 00 CET and 20 : 00 local time in Tbilisi)

## How to join:

The seminar is organized on the **Cisco Webex Meetings**. If you are already registered, you do not need to register again. Otherwise, to join the seminar please send an e-mail to [seminarim@ug.edu.ge](mailto:seminarim@ug.edu.ge) or register here:

<https://forms.gle/xfQJ9fg1uqe7CrZw6>

You will then receive further information.

**WEB of Seminar:** <https://www.ug.edu.ge/en/tbilisi-analysis-and-pde-seminars>

**Organizers:**

1. R. Duduchava, Institute of Mathematics, University of Georgia, Tbilisi
2. E. Shargorodsky, Department of Mathematics, King's College London
3. G. Tephnadze, Institute of Mathematics, University of Georgia, Tbilisi



**Secretary:**

M. Tsaava, Institute of Mathematics, University of Georgia, Tbilisi

**Technical support:**

Z. Vashakidze, Institute of Mathematics, University of Georgia, Tbilisi