



Dear Colleagues!

V. Kupradze Institute of Mathematics of the University of Georgia is pleased to invite you to the Online Tbilisi Analysis & PDE Seminar. The seminar is held bi-weekly on Tuesdays at 20 : 00 local time in Tbilisi.

Talk on May 13, 2025

Speaker: Prof. David Cruz-Urbe OFS, The University of Alabama.
<https://math.ua.edu/people/david-cruz-uribe/>

"Bounded solutions of degenerate elliptic equations with an Orlicz-gain Sobolev inequality"

Abstract: It is a classical result, due to Trudinger and others, that if Q is an $n \times n$, self-adjoint, uniformly elliptic matrix function, then the Dirichlet problem

$$\begin{cases} -\operatorname{Div}(Q\nabla u) = f, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

has a bounded solution provided that $f \in L^q(\Omega)$, $q > n/2$. I will discuss recent work with Scott Rodney where we considerably generalize this result. Earlier, we showed that we could replace the assumption that $f \in L^q$ with the assumption that f was an element of the Orlicz space $L^A(\Omega)$, where $A(t) = t^2 \log(e + t)^\sigma$ and $\sigma > n/2$.

In our current work, we consider the boundedness and exponential integrability of solutions to the Dirichlet problem for the degenerate elliptic equation

$$\begin{cases} -v^{-1} \operatorname{Div}(|\sqrt{Q}\nabla u|^{p-2} Q\nabla u) &= f|f|^{p-2} \text{ in } \Omega, \\ u &= 0 \text{ in } \partial\Omega. \end{cases}$$

Of particular interest is the case when Q is not uniformly elliptic and there exist functions w and v , not necessarily bounded or bounded away from 0, such that for all $\xi \in \mathbb{R}^n$,

$$w(x)|\xi|^p \leq |\sqrt{Q}\xi|^p \leq v(x)|\xi|^p.$$

Following the approach of Sawyer and Wheeden, we prove our results in an abstract setting, where we assume the existence of a Sobolev inequality of the form

$$\|\varphi\|_{L^N(v, \Omega)} \leq S_N \|\sqrt{Q}\nabla \varphi\|_{L^p(\Omega)}$$

and N is a power function of the form $N(t) = t^{\sigma p}$, $\sigma \geq 1$, or a Young function of the form $N(t) = t^p \log(e + t)^\sigma$, $\sigma > 1$. We study the interplay between the regularity of the function f , expressed in terms of membership in an Orlicz space, the gain in the Sobolev inequality, and the boundedness or exponential integrability of the solution.

If there is time, at the end I will briefly discuss work with my graduate student, Feyza Elif Dal, where we show how to construct specific examples of the Sobolev inequalities that we assume.

Date: May 13, 2025

Time: 20 : 00 local time in Tbilisi;

(Compare to your local time: <https://www.timeanddate.com/worldclock/georgia/tbilisi/>);

How to join:

The seminar is organized on the **Zoom Platform**. If you are already registered, you do not need to register again. Otherwise, to join the seminar please send an e-mail to seminarim@ug.edu.ge or register here:

<https://forms.gle/xfQJ9fg1uqe7CrZw6>

You will then receive further information.

WEB of Seminar: <https://www.ug.edu.ge/en/tbilisi-analysis-and-pde-seminars>

Organizers:

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2. E. Shargorodsky, Department of Mathematics, King's College London
3. G. Tephnadze, Institute of Mathematics, University of Georgia, Tbilisi



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